XXIII. *On the Nature of the Sun's Magnetic Action upon the Earth.*

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1. IN attempting to frame a theory which shall account for the relations which have been shown to exist between the variations of terrestrial magnetism and the position of the sun with respect to the place of observation on the earth's surface, the following question presents itself for consideration at an early stage of the inquiry, "Are the magnetic effects produced on the earth such as could be explained by the simple supposition that the sun is a great magnet, or not?" The solution of this question will, to a certain extent, limit the range of probable sources from which to seek the true cause of magnetic variations, and is therefore worthy of attention.

2. In the first place, let us endeavour to find the *law of the diurnal variations* of the Declination, Horizontal Force, and Vertical Force at a given place on the earth's surface, on the supposition that these variations arise from the varying relations, as to position, of the sun acting as a magnet upon the earth.

3. The sun would affect the magnets used for showing the earth's changes of force in two ways—first directly, and secondly by inducing magnetism in the soft iron and other inductive matter forming part of the body of the earth, this induced magnetism reacting upon the observed magnets.

4. Now this subject has been discussed mathematically by POISSON in the case of masses of soft iron having any possible arrangement, and he has given expressions for the combined effect of the direct and induced forces upon the magnets, making only this restriction, which is allowable in the case under consideration, that the length of the magnets must be infinitesimally small in comparison with their distance from the nearest particle of iron. The expressions are as follows:—

$$X'=(1+a)X+bY+cZ, \qquad (a.)$$

$$Y'=dX+(1+e)Y+fZ, \qquad (b.)$$

$$Z'=gX+hY+(1+k)Z, \qquad (c.)$$

where, assuming the direction and intensity of the magnetic force exerted directly by the sun upon the centre of the earth to be approximate values of the same elements throughout the body of the earth, X represents the resolved part of that force along the perpendicular to the earth's axis from the place of observation, being reckoned positive when the resolved force acting alone would make the north end of a magnet at the station point towards the earth's axis; Y is the resolved part of the same force perpendicular to the meridian plane, the positive direction being to eastward; Z represents the part of the force resolved along the earth's axis, plus to northward, and a, b, c, d, e, f, g, h, and k are constants which depend on the amount and arrangement of the soft iron of the earth. X$'$, Y$'$, and Z$'$ are the combined effects of the direct force of the sun, and of the reaction of the magnetism which it induces in the earth, upon the magnets at the place of observation, in the directions of X, Y, and Z respectively.

POISSON's equations * may be derived from the following considerations. The inducing force X produces a certain magnetic state in the mass of iron, and the magnetism thus induced (including that which is caused by the inducing action of the particles of iron upon each other) will exert an attraction at the place of observation in the direction of each of the coordinate axes, which attraction is assumed to be proportional to the inducing force X; the forces due to X in the directions of X, Y, and Z respectively will therefore be aX, dX, and gX, and those due to Y, bY, eY, hY; those due to Z, cZ, fZ, kZ. Now, adding the forces in the same directions and including the forces X, Y, and Z, which act directly upon the magnets at the place of observation, we get

$$X'=(1+a)X+bY+cZ,$$
$$Y'=dX+(1+e)Y+fZ,$$
$$Z'=gX+hY+(1+k)Z.$$

5. In order that POISSON's formulæ should be strictly applicable to the case under consideration, we must assume that the whole effect of an inducing force acting upon the earth requires only a few minutes to be acquired, and that it is lost with equal facility on the removal of the inducing force. It is not necessary that the effects of gain and loss should be momentary.

6. We must now transform the coordinates of POISSON's equations to those directions in which the variations of the earth's magnetism are always observed, viz. towards the *magnetic* east (x), towards the *magnetic* north (y), and vertically downwards (z), the forces being reckoned positive when, acting alone, they would make the north end of a magnet point in the directions named, and we get a new set of equations,

$$x=AX+BY+CZ, \quad \ldots \quad (d.)$$
$$y=DX+EY+FZ, \quad \ldots \quad (e.)$$
$$z=GX+HY+KZ, \quad \ldots \quad (f.)$$

in which A, B, C, ... K are constant numbers, being functions of a, b, c, ... k, and of the magnetic declination and latitude of the place of observation.

7. Let us observe that, during the time in which the earth is making a single revolution upon its axis, the variations in direction and amount of Z and of R (the resultant of X and Y) may be disregarded, while the direction and value of X and Y vary with the hour-angle (h) of the sun, and may be expressed as follows:—

$$X=R\cos(h+\alpha), \quad \ldots \quad (g.)$$
$$Y=R\sin(h+\alpha), \quad \ldots \quad (h.)$$

where $\tan\alpha=\frac{Y_0}{X_0}$, . . . (*i.*) and $R^2=X_0^2+Y_0^2$, ... (*k.*)

* This mode of obtaining POISSON's formulæ was suggested by Mr. ARCHIBALD SMITH, F.R.S.

X_0 and Y_0 being the values of X and Y *at noon*. It is evident that, Z being considered constant, the last terms of (*d.*), (*e.*), (*f.*) form no part of the diurnal variations, and may be neglected; omitting them therefore and substituting the values of X and Y given by (*g.*) and (*h.*), the equations (*d.*), (*e.*), and (*f.*) become

$$\left.\begin{aligned} x'_h &= \mathrm{R}\{\mathrm{A}\cos(h+\alpha)+\mathrm{B}\sin(h+\alpha)\} \\ &= \mathrm{R}\sqrt{\mathrm{A}^2+\mathrm{B}^2}\sin\{(h+\alpha)+\beta\}, \\ &\quad \text{where } \tan\beta=\frac{\mathrm{A}}{\mathrm{B}}; \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (l.)$$

$$\left.\begin{aligned} y'_h &= \mathrm{R}\{\mathrm{D}\cos(h+\alpha)+\mathrm{E}\sin(h+\alpha)\} \\ &= \mathrm{R}\sqrt{\mathrm{D}^2+\mathrm{E}^2}\sin\{(h+\alpha)+\beta_{\prime}\}, \\ &\quad \text{where } \tan\beta_{\prime}=\frac{\mathrm{D}}{\mathrm{E}}; \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (l'.)$$

$$\left.\begin{aligned} z'_h &= \mathrm{R}\{\mathrm{G}\cos(h+\alpha)+\mathrm{H}\sin(h+\alpha)\} \\ &= \mathrm{R}\sqrt{\mathrm{G}^2+\mathrm{H}^2}\sin\{(h+\alpha)+\beta_{\prime\prime}\}, \\ &\quad \text{where } \tan\beta_{\prime\prime}=\frac{\mathrm{G}}{\mathrm{H}}. \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (l''.)$$

As the angles α, β, $\beta_{\prime}$, and $\beta_{\prime\prime}$ may be regarded as constant for a single day, the equations (*l.*), (*l'.*), (*l''.*) show that the deviations of the magnets from their normal positions at any two hours whose interval is half a day are equal in amount but opposite in direction for each element of the earth's force; but this is quite at variance with the real character of the diurnal variations, the excursions of the magnets being invariably extensive on both sides of their normal positions during about eight hours of the day, which include the sun's upper transit, while deviations of comparatively small extent occur during the remaining hours.

This discordance will be more conclusively shown if we express the observed variations in the form of the series (*y*) described hereafter (17), when the coefficients of the second and following terms should be zero, while we find that those of the second and third terms have magnitudes not greatly inferior to the coefficient of the first term.

Again, it will be shown hereafter (19) that there should be no mean diurnal variation *for the whole year* due to direct action of the sun, while in fact there is such a variation of very considerable range, in middle and high magnetic latitudes, which is opposite in character for the north and south magnetic hemispheres.

With respect to the direct action of the sun upon the observed magnets, the first of the above-mentioned reasons was adduced by Dr. LLOYD as showing the inconsistency with observation in the law of the diurnal variations as derived from the theory in question, in a paper published in the 'Proceedings' of the Royal Irish Academy, February 22, 1858. The same conclusion is here extended by the aid of POISSON's formula to the inducing action of the sun upon the soft iron of the earth. The hypothesis of the magnetic nature of the sun cannot, therefore, by itself be accepted as an explanation of the diurnal variations of the earth's magnetism.

9. It may be argued, however, that the diurnal variations are the combined effects of direct solar action and of other forces whose origin and nature are unknown; but if this be the case, I think we have reason for believing that the portion of the variations which proceeds from the former source is small in comparison with the part which is due to the other forces that are in operation.

10. Returning now to the equation ($l.$), if R_0 and α_0 be any particular values of R and α, and any other values be R_γ and $(\alpha_0+\gamma)$, we shall have in the latter case

and if $\beta'=\alpha_0+\beta$,

$$\left.\begin{aligned} x'_h &= R_\gamma\sqrt{A^2+B^2}\sin(h+\alpha_0+\gamma+\beta), \\ &= R_\gamma\sqrt{A^2+B^2}\sin(h+\gamma+\beta'). \end{aligned}\right\} \quad \ldots \ldots \quad (p.)$$

Hence it appears that when R_0 and α_0 are changed to R_γ and $(\alpha_0+\gamma)$, the range of variation is altered in the proportion of R_γ to R_0, and the hour at which the maximum deviation occurs is advanced by the equivalent in time to the angle γ.

As we are only seeking the law of variation and not absolute quantities, we may reject the constant factor $\sqrt{A^2+B^2}$, when ($p.$) becomes

$$x''_h = R_\gamma \sin(h+\gamma+\beta'). \quad \ldots \ldots \ldots \quad (t.)$$

12. Now let us observe that the period embraced by a large mass of hourly observations, extending over several years, comprises many revolutions of the sun on its axis, the sun's period of rotation being about twenty-five days, and that it is from such extended series of observations that the determinations of the regular diurnal variations, which we shall use for comparison with the results of our hypothesis, are derived.

13. As the distance of the sun from the earth is great in relation to the diameter of the sun, we may assume, in the calculation of the magnetic force exerted by the sun upon the earth, that the sun's free magnetism is accumulated in the extremities of a straight line whose middle point passes through the sun's centre; and this supposed magnet may be resolved into two others, one having its poles in the sun's axis of rotation, the other with its poles in the plane of the sun's equator. Now as regards the latter, if we notice its effect upon the earth when its north end points towards the earth, it is evident that when the sun has made somewhat more than half a revolution upon its axis, the south end will point towards the earth and the opposite effect be produced, and similarly, for every position of this magnet there is another position, when the sun has advanced about 180° in rotation, in which the effect neutralizes that of the former position.

14. It is necessary therefore to consider only the magnetism of the sun parallel to its axis of rotation as affecting the mean diurnal variations on the earth's surface. Now we have no means of finding the absolute magnetic force of the sun, neither do we require it; what we want is the *law of change* of direction and intensity of the force exerted by the sun upon the centre of the earth as the earth revolves in its orbit; and this we can easily calculate from the known elements of position of the sun's axis, which by

Mr. Carrington's latest determinations are—

Inclination of sun's axis to plane of ecliptic (i)$=82°\ 45'$,

Longitude of projection of northern half of sun's axis on plane of ecliptic

at epoch 1850·0 (λ')$=343°\ 40'$.

at epoch 1845·0 (λ')$=343°\ 35'$.

(See Monthly Notices of the Royal Astronomical Society, vol. xxii. page 300.)

15. Let I be the magnetic moment of the sun in the direction of its axis of rotation, supposed positive if the north end of the sun's axis is a north pole;

D the radius vector of the earth;

ω the obliquity of the ecliptic;

θ the right ascension of the sun;

λ the longitude of the sun;

δ the declination of the sun, positive when north;

and δ' and θ' the declination and right ascension which the sun had when his longitude was $(\lambda-90°)$.

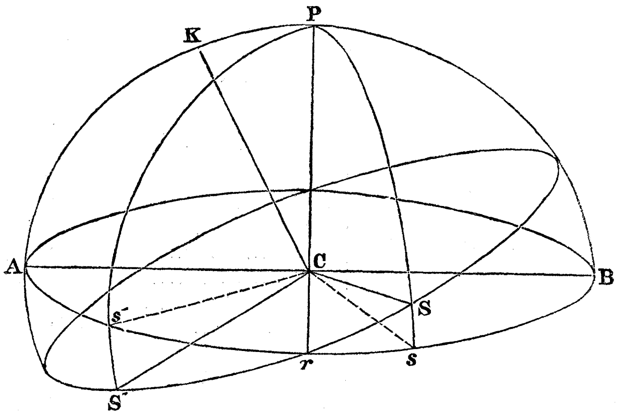
Now, if we suppose the magnetism of the sun along its axis to be resolved perpendicular to the plane of the ecliptic (+ to northward) $=\mathrm{I}\sin i$, and in that plane, and the latter part to be again resolved in the directions of the line joining the centres of the sun and earth (+ towards the earth)$=-\mathrm{I}\cos i\cos(\lambda-\lambda')$, and at right angles to that line (+ to the right hand of an observer who looks towards the earth, his head being supposed at the north pole and feet at south pole of the sun)$=-\mathrm{I}\cos i\sin(\lambda-\lambda')$, then the forces which these exert on the centre of the earth in the same directions respectively, are

$$\mathrm{L}=-\frac{\mathrm{I}}{\mathrm{D}^3}\sin i,$$

$$\mathrm{M}=-2\frac{\mathrm{I}}{\mathrm{D}^3}\cos i\cos(\lambda-\lambda'),$$

$$\mathrm{N}=+\frac{\mathrm{I}}{\mathrm{D}^3}\cos i\sin(\lambda-\lambda').$$

Now let us change the coordinates to those of X_0, Y_0, and Z; this will be facilitated by a reference to the accompanying figure, where C represents the centre of the earth, ArB the equator, and P its north pole, S$'r$S the ecliptic, and K its pole, and S the sun; then



$r\mathrm{CS}=\lambda$,

$r\mathrm{C}s=\theta$,

$\mathrm{SCS}'=90°$,

$s\mathrm{C}s'=(\theta-\theta')$,

$s\mathrm{CS}=\delta$,

$s'\mathrm{CS}'=\delta'$,

$\mathrm{PCK}=\omega$.

The resolved part of L along $Z = -\frac{I}{D^3} \sin i \cos \omega$,

The resolved part of M along $Z = +2\frac{I}{D^3} \cos i \cos(\lambda - \lambda') \sin \delta$,

The resolved part of N along $Z = -\frac{I}{D^3} \cos i \sin(\lambda - \lambda') \sin \delta'$,

and the whole force

$$Z = -\frac{I}{D^3}\left\{\sin i \cos \omega - 2\cos i \cos(\lambda - \lambda') \sin \delta + \cos i \sin(\lambda - \lambda') \sin \delta'\right\}.$$

The resolved part of L along $X_0 = -\frac{I}{D^3} \sin i \sin \omega \sin \theta$,

The resolved part of M along $X_0 = -2\frac{I}{D^3} \cos i \cos(\lambda - \lambda') \cos \delta$,

The resolved part of N along $X_0 = +\frac{I}{D^3} \cos i \sin(\lambda - \lambda') \cos \delta' \cos(\theta - \theta')$,

and the whole force

$$X_0 = -\frac{I}{D^3}\left\{\sin i \sin \omega \sin \theta + 2\cos i \cos(\lambda - \lambda') \cos \delta - \cos i \sin(\lambda - \lambda') \cos \delta' \cos(\theta - \theta')\right\} \quad (u.)$$

The resolved part of L along $Y_0 = +\frac{I}{D^3} \sin i \sin \omega \cos \theta$,

The resolved part of M along $Y_0 = 0$,

The resolved part of N along $Y_0 = +\frac{I}{D^3} \cos i \sin(\lambda - \lambda') \cos \delta' \sin(\theta - \theta')$,

and the whole force $Y_0 = +\frac{I}{D^3}\left\{\sin i \sin \omega \cos \theta + \cos i \sin(\lambda - \lambda') \cos \delta' \sin(\theta - \theta')\right\} \quad . \quad (v.)$

16. Taking 1845 as a convenient year, being about the middle year of the observations which we shall make use of, and calculating by means of the formulæ (*u.*), (*v.*), and data taken from the Nautical Almanac, the values of X_0 and Y_0 for the middle day of each month of the year, we obtain the following results, which it may be remarked would be altered only in a trifling degree if any year between 1835 and 1855 were taken instead of 1845.

TABLE I.

	X_0.	Y_0.
	I ×	I ×
January	+ ·20618	+ ·08202
February	− ·01627	+ ·29425
March	− ·21637	+ ·41678
April	− ·33852	+ ·44028
May	− ·33349	+ ·35721
June	− ·34793	+ ·16582
July	− ·21580	− ·06992
August	− ·00104	− ·28116
September	+ ·20327	− ·41070
October	+ ·32934	− ·44304
November	+ ·37919	− ·36388
December	+ ·34916	− ·17132

The constant I multiplies all these numbers, and therefore has no influence upon the law of change; hence it may be omitted, and we shall then call the numbers $X'_0\left(=\frac{X_0}{I}\right)$ and $Y'_0\left(=\frac{Y_0}{I}\right)$, the resultant of X'_0 and Y'_0 being called R'_γ.

By means of the above Table and equations (*g.*), (*h.*), and (*i.*), we are enabled to construct a table of values of R'_γ and α, when, calling α_0 the January value of α, γ is found by subtracting α_0 successively from the α of each month.

TABLE II.

	R'_γ.	α.	γ.
		° ′	° ′
January	·22190	21 42	0 0
February	·29479	93 10	71 28
March	·46960	117 26	95 44
April	·55537	127 33	105 51
May	·48869	133 2	111 20
June	·38543	154 31	132 49
July	·22686	197 58	176 16
August	·28116	269 47	248 5
September	·45824	296 20	274 38
October	·55204	306 38	284 56
November	·52555	316 11	294 29
December	·38892	333 52	312 10

If in equation (*t.*) x''_h become x'''_h when R_γ becomes R'_γ, we obtain, by inserting the above values of R'_γ and γ in

$$x'''_h = R'_\gamma \sin\{(h+\gamma)+\beta'\}, \quad . \quad . \quad . \quad . \quad (w.)$$

the following equations showing the diurnal variations in the different months:

$$\left.\begin{array}{lll} \text{January} & . \; . \; . & x'''_h = \cdot 22190 \sin\ (h+\beta'), \\ \text{February} & . \; . \; . & x'''_h = \cdot 29479 \sin\{(h+71^\circ\ 28')+\beta'\}, \\ \text{March} & . \; . \; . & x'''_h = \cdot 46960 \sin\{(h+95^\circ\ 44')+\beta'\}, \\ \quad \text{\&c. \&c.} & & \end{array}\right\} \quad . \quad . \quad . \quad . \quad (x.)$$

It will be noticed that the angle β' is undetermined, depending as it does upon the distribution of soft iron in the earth as well as upon the angle α_0; but it is not required in the application that we are to make of the formulæ.

17. Let us now turn our attention to the results of observation as to the diurnal variations. General SABINE has discussed this subject very fully for the observations made at the British Colonial Observatories of Toronto, St. Helena, Hobarton, &c. After a careful separation of disturbed observations, he has given hourly mean values of the deviations of the observed magnets from their normal positions, or their equivalents expressed in terms of the earth's force, for every month of the year, the means being deduced from several years' observations.

Now, if from the mean hourly deviations for a given month we determine by the method of least squares the constants in a series of the form

$$\Delta_h = \text{B}\sin(h+\delta)+\text{C}\sin 2(h+\varepsilon)+\text{D}\sin 3(h+\iota)+\&\text{c.}, \quad \ldots \quad (y.)$$

where Δ_h is the deviation at any hour (h) of the day, it is easy to show that the values of the constants in any one term are independent of the values of those in any other term, if the data are complete for all the twenty-four hours, and if the series be not carried beyond a term $\text{W}\sin 23(h+\zeta)$.

18. We have proved (16) that the variation due to direct and inducing action of the sun is of the form

$$x_h'''' = \text{R}_\gamma' \sin\{(h+\gamma)+\beta'\}, \quad \ldots \ldots \quad (\omega.)$$

and therefore we see that the first term of the series ($y.$) includes the whole of this action, the remaining terms being unaffected by it. Moreover the equations ($x.$) show us that the part of the term $\text{B}\sin(h+\delta)$ which is due to direct solar action varies as R_γ', and has its maximum advanced as γ increases. Thus we are led to expect changes in B and δ from month to month which shall accord with the corresponding variations of R_γ' and γ, and the degree of accordance which is found to exist will serve to indicate the extent to which the direct and inducing action of the sun bears a part in the production of the regular diurnal variations.

19. We see from Table II. that the values of R_γ' for January and July are nearly equal, and that the angle γ for the former month differs by about 180° from its value for the latter month: consequently there should be no *mean* diurnal variation (or a very small one) for these two months, inasmuch as the hour of maximum deviation in the one is the same as the hour of minimum in the other, the magnitude of the deviations being nearly alike in the two months: the same observation holds with regard to any two months separated by an interval of half a year, and thus we perceive that the *mean* diurnal variation *for the whole year*, due to direct action of the sun, should be extremely small, and that, without sensible error, we may omit the consideration of it.

We may therefore separate from the quantity $\text{B}\sin(h+\delta)$ for the different months, its normal value for the year, calling the remaining quantities $\text{B}'\sin(h+\delta')$; and we shall then have to consider only the latter numbers for comparison with R_γ' and γ, for the reason just stated.

Now it is probable (from the simplicity of the instrument used and its independence of temperature corrections) that no periodical magnetic variations are so well determined as the diurnal variations of declination. We shall therefore confine ourselves to the examination of the variations of that element, for which the following Table gives the monthly values of B′ and δ' at Toronto and St. Helena, these being derived from tables of variations given by General SABINE in his discussions of the observations made at the observatories of Toronto and St. Helena, published for the British Government by LONGMAN and Co., London. Positive values of $\text{B}'\sin(h+\delta')$ indicate easterly deviations of the north end of the declination magnet.

TABLE III.

	Toronto.		St. Helena.	
	B′.	δ′.	B′.	δ′.
	′	° ′	′	° ′
January	1·208	24 20	·627	349 48
February	·883	35 44	·916	347 8
March	·467	188 22	·570	353 46
April	·664	228 21	·203	196 1
May	·884	204 14	·640	192 0
June	1·053	179 44	·858	174 35
July	1·006	177 6	·922	175 24
August	·963	207 23	·803	183 21
September	·569	261 12	·372	174 8
October	·444	17 49	·642	50 11
November	1·095	14 49	·688	4 40
December	1·472	17 26	·603	348 8
	B.	δ.	B.	δ.
Year	2·625	32 37	·126	292 29

20. As a diagram conveys to the mind a more distinct conception of a variation than a table of numbers, the curve (Plate XXV. fig. 1) is constructed to represent the successive values of R'_γ and γ: in this figure the lines $r\,1, r\,2, \ldots r\,12$ are proportional to R'_γ, and the angles $A\,r\,1, A\,r\,2, \ldots A r 12$ are equal to the angle γ, for January, February, . . . December respectively. The curves of figs. 2 & 3, which are formed in a similar manner from the numbers in the preceding Table, are intended to show the variations of B′ and δ'. Drawing lines $r\,G'$, $r\,H'$ at right angles to $r\,G$ and $r\,H$ respectively, it is easy to see, from the form of the expression $B' \sin(h+\delta')$, that the angles $G' r\,1$, $G' r\,2$, . . . $G' r 12$ (reckoned by a right-handed revolution from $G' r$) represent the hour-angles of the sun at the time of the occurrence of the maximum deviation in the respective months from January to December, and that the extent of that deviation is represented by the lines $r\,1, r\,2, \ldots r\,12$ respectively; and similarly for fig. 3.

21. Now we see that direct action of the sun is not the sole cause of the variations of the term $B \sin(h+\delta)$, because in that case fig. 1 (which represents the variations of the cause) would be similar to figs. 2 & 3 (which represent the variations of the effect), but would not be similarly situated unless the angle β' happened to be equal to $G\,r\,1$ or $H\,r\,1$; and we find but little appearance of similarity displayed by the curves; the extent of the likeness will be exhibited more distinctly in what follows. We may here state that if the quantity I were negative, it would only have the effect of increasing β' by 180°.

22. The lines C D, E F in figs. 2 & 3 have that direction which gives the sum of the squares of the perpendiculars let fall upon them from the points 1, 2, . . . 12 a minimum, and it is very noticeable that the points are arranged in closer proximity to these lines than to lines at right angles to C D and E F, the points 10, 11, 12, 1, 2 being

generally towards one extremity of the lines, while the remaining points are towards the opposite extremity.

The expression $B' \sin(h+\delta')$ may be written

$$B' \cos(\delta'-\sigma) \sin(h+\sigma)+B' \sin(\delta'-\sigma) \cos(h+\sigma),$$

where, σ being the angle $G r D$ or $H r F$, the former term represents that part of the variation of the term $B \sin(h+\delta)$ which gives a maximum of easterly declination when $h=90°-\sigma$, its coefficient being the resolved part of the lines $r\,1$, $r\,2$, . . . $r\,12$ along C D or E F; and the latter term represents the part of the variation of $B \sin(h+\delta)$ which gives a maximum of declination when $h=-\sigma$, and its coefficient is the resolved part of $r\,1$, $r\,2$, . . . $r\,12$ at right angles to C D or E F. The following Table shows the values of $B' \cos(\delta'-\sigma)$ and $B' \sin(\delta'-\sigma)$ for the different months:—

TABLE IV.

	Toronto.		St. Helena.	
	$B' \cos(\delta'-\sigma)$.	$B' \sin(\delta'-\sigma)$.	$B' \cos(\delta'-\sigma)$.	$B' \sin(\delta'-\sigma)$.
January	+1·196	+·172	+·620	−·094
February	+ ·832	+·296	+·898	−·179
March	− ·463	+·063	+·569	−·046
April	− ·562	−·354	−·193	−·061
May	− ·875	−·124	−·622	−·150
June	−1·010	+·298	−·856	+·057
July	− ·951	+·328	−·921	+·048
August	− ·944	−·188	−·800	−·069
September	− ·240	−·516	−·371	+·028
October	+ ·444	+·013	+·397	+·504
November	+1·094	−·045	+·684	+·075
December	+1·471	+·033	+·593	−·107

In figs. 5, 6, 7, & 8 (Plate XXVI.) the monthly variation of these numbers is indicated by curves whose vertical ordinates are proportional to the numbers, positive values being reckoned upwards; and the abscissæ are divided into twelve equal parts to represent the successive months of the year.

23. Remembering what was said in art. 19, we see that if a straight line be drawn *in any direction* through the point r of fig. 1, it divides the curve into two sets of six consecutive months; and that if perpendiculars be drawn from the points 1, 2, 3, . . . 12 to that line, their lengths are least in the extreme months on each side of the line, and increase to maxima and then diminish as the numbers increase, there being only one maximum on each side. Now as this is true of any line, these conditions must be fulfilled (whatever be the value of the angle β') by the ordinates of figs. 5, 6, 7, & 8 if the variations which the curves represent are due to direct action of the sun: and if we confine our attention to figs. 5 & 7, these conditions are satisfied in a remarkable manner; but as we have found that they are to be demanded alike from figs. 6 & 8

before we can admit the truth of the hypothesis, let us see whether these curves also exhibit the required form.

24. In the first place, we observe that the range of these curves is much smaller than that of figs. 5 & 7, and still smaller than the whole diurnal range of declination at the two stations, and therefore, even if the curves had the required form, we should still have reason to believe that the direct effect of the sun was less than that of other forces in operation. For the ratio of the mean ordinate (disregarding signs) of fig. 6 is to that of fig. 5 as 1 to 4·2, and the corresponding ratio for figs. 8 & 7 is as 1 to 5·3, while the least possible ratio of the mean ordinates upon rectangular diameters of fig. 1 is as 1 to 2·7; and therefore, under the most favourable supposition as to the value of the angle β', the mean ordinates of figs. 5 & 7 should be considerably less, or those of figs. 6 & 8 greater, in order that the results of observation should accord with the hypothesis. But figs. 6 & 8 are far from fulfilling the conditions named as regards the form of the curves; for we see that neither of them has but a single maximum or a single minimum ordinate, nor are six consecutive months above and six below the horizontal line deviating most from that line in the middle months of each group of six.

The general appearance of figs. 6 & 8 conveys the impression that the numbers which they represent are possibly errors in the determinations of the respective values of B$'$ and δ', arising perhaps from the uneliminated part of the disturbances; but if it were possible to decompose them into two parts, one obeying the required law of variation and another following some other law, it is probable that the mean ordinate corresponding to the former part would be much less than the mean of the combined ordinates: hence it is probable that there is but little trace of direct solar action to be found in the regular diurnal variations of declination at Toronto and St. Helena.

25. It may be objected, however, that the agreement of figs. 5 & 7 with the requirements of the hypothesis does not appear as well in figs. 6 & 8, because the other variable forces in operation act in partial opposition to the sun, and so mask the character which figs. 6 & 8 would have if those opposing forces could be separated. To this objection it will be sufficient to reply that the variations of the quantities corresponding to $B'\cos(\delta'-\sigma)$ for the second and third terms of the series (y) have a similar character to those of the first term, and it appears reasonable therefore to infer that the same variable force is the cause of the variations of each of the three terms; but the second and third terms have been proved (18) to be independent of direct action of the sun; hence it is most probable that the variations of $B'\sin(\delta'-\sigma)$, though they accord with the hypothesis of direct solar action, are due to some different cause. The similarity of the variations of $B'\cos(\delta'-\sigma)$ is shown by the curves of figs. 9 & 11 (Plate XXVII.) in continuous black, red, and interrupted black lines, which have reference to the first, second, and third terms of (y) respectively. Figs. 10 & 12 are similar representations of the variations of $B'\sin(\delta'-\sigma)$ for the first three terms of (y); and it is observable that these are quite different one from another. In Table V. are given the numbers from which the curves of figs. 9, 10, 11, & 12 are constructed.

TABLE V.—Showing the variations of the quantities corresponding to $B' \cos(\delta'-\sigma)$ and $B' \sin(\delta'-\sigma)$ for the second and third terms of the series (y).

	Toronto.				St. Helena.			
	For second term.		For third term.		For second term.		For third term.	
	$B' \cos(\delta'-\sigma)$.	$B' \sin(\delta'-\sigma)$.	$B' \cos(\delta'-\sigma)$.	$B' \sin(\delta'-\sigma)$.	$B' \cos(\delta'-\sigma)$.	$B' \sin(\delta'-\sigma)$.	$B' \cos(\delta'-\sigma)$.	$B' \sin(\delta'-\sigma)$.
January ...	+ ·919	−·020	+·497	+·026	+·409	+·196	+·345	+·103
February...	+ ·868	−·096	+·429	−·195	+·503	−·604	+·235	−·461
March......	+ ·399	+·446	+·120	+·371	+·417	−·488	+·315	−·340
April	− ·184	+·145	−·014	+·138	+·050	−·069	+·232	−·041
May	− ·856	−·126	−·371	−·134	−·560	−·081	−·360	+·089
June	− ·875	+·281	−·295	−·070	−·543	+·098	−·533	+·135
July	− ·574	+·375	−·370	+·012	−·602	+·074	−·614	+·125
August ...	−1·212	+·061	−·751	+·018	−·825	+·012	−·690	+·191
September	− ·697	−·714	−·459	−·049	−·479	−·045	−·307	−·010
October ...	+ ·499	−·314	+·139	−·007	+·624	+·207	+·506	−·082
November	+ ·643	−·029	+·374	−·092	+·568	+·298	+·442	+·094
December	+1·075	−·006	+·694	−·017	+·440	+·398	+·457	+·116

An important inference that may be drawn from the curves of figs. 9, 10, 11, & 12, is that, as those of figs. 9 & 11 exhibit a remarkable likeness of character, while those of figs. 10 & 12 present little appearance of similarity, the former are probably representatives of true changes, while the latter may perhaps be regarded as errors in the determinations, when we obtain the following approximate expression for the variable part of the diurnal variations in any month of the year,

$$A\{B \sin(h+\alpha)+C \sin 2(h+\beta)+D \sin 3(h+\gamma)+\&c.\},$$

where A alone varies from month to month, being sometimes positive and sometimes negative. If we might regard this as a true statement, it would follow, as the law of variation remains constant throughout the year, the range only altering, that these variations do not result from those of different forces, but from those of a single force, or of a group of connected forces.

26. Having seen that it is probable that if the sun be a magnetic body its direct magnetic influence upon the earth, as evinced by the regular diurnal variations, is very small, I think we may safely infer that terrestrial magnetic disturbances are not caused by variations in the magnetism of the sun; for if the whole direct action of the sun (at ordinary times) be insensible, much more will the changes which take place in the sun's magnetic state be insensible in their effects upon the earth, as it would be difficult to conceive that the changes in question were ever of such magnitude as to approach the effect of reversing the sun's ordinary magnetic condition.

27. I conclude, therefore, that the mode in which forces originating in the sun influence the magnetic condition of the earth is not analogous to the action of a magnet upon a mass of soft iron placed at a great distance from it, but that these forces proceed from the sun in a form different from that of magnetic force, and are converted into the latter form of force probably by their action upon the matter of the earth or its atmosphere.

On the existence of a variation of diurnal range of Declination at St. Helena depending on the Sun's altitude.

When we express the diurnal variations of declination at St. Helena for the different months of the year in the form (*y*),

$$\Delta_h = \text{B} \sin (h+\delta) + \text{C} \sin 2(h+\varepsilon) + \text{D} \sin 3(h+\iota),$$

the values of the coefficients B, C, D are seen to be larger about the time when at noon the sun is overhead at St. Helena, which occurs in the first weeks of February and November, than they are immediately before or after. This fact indicates the existence of a secondary variation of the diurnal range of declination which has maximum values at the times mentioned, and which is additional to the variation of range caused by the semiannual inequality discovered by General SABINE. The monthly values of B, C, and D are shown in the following Table:—

	B.	C.	D.
January	′·703	′·726	′·622
February	·994	1·043	·968
March	·640	·904	·904
April	·226	·375	·619
May	·629	·292	·352
June	·808	·243	·428
July	·872	·299	·499
August	·771	·524	·534
September	·331	·204	·427
October	·594	·939	·857
November	·737	·903	·713
December	·682	·817	·663

Had this been a feature of only one or even two of the three coefficients, we might have supposed it to be an accidental circumstance which would perhaps be reversed on a repetition of the observations; but as the three coefficients agree in giving the same testimony, I think we must allow it to be not merely an apparent but a real variation.

Before concluding, I must acknowledge the important help that I have derived from Mr. ARCHIBALD SMITH'S 'Mathematical Theory of the Errors of Ship's Compasses,' of which what has preceded is little more than an application to another purpose. I am also indebted to Mr. THOMAS W. BAKER, of the Kew Observatory, for the very efficient assistance which he has afforded me in the rather laborious computations involved in the discussion.

Note on §§ 26 and 27. By Professor W. THOMSON, *M.A., LL.D., F.R.S.*

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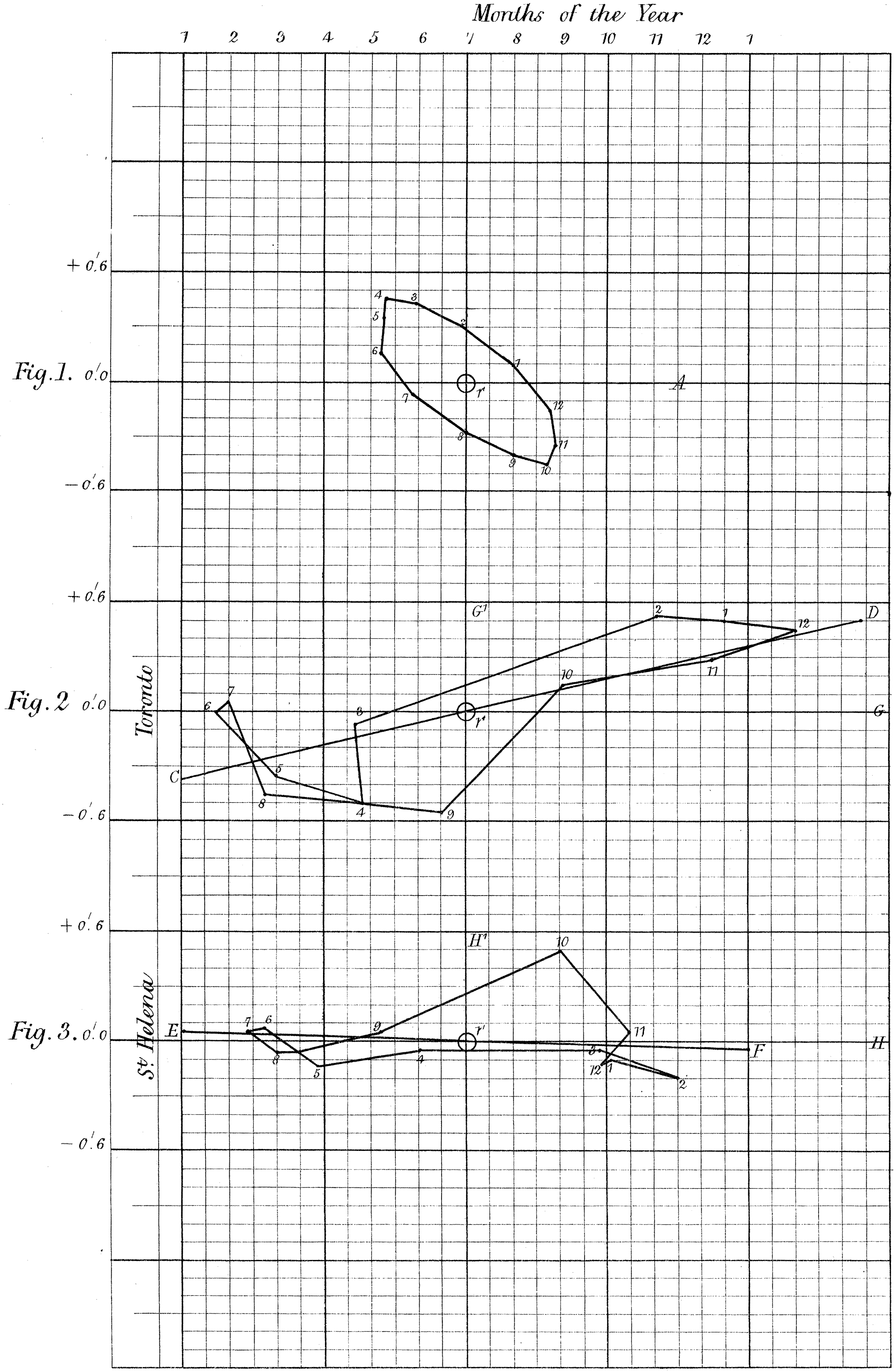
If the sun were a magnet as intense on the average as the earth, the magnetic force it would exert at a distance equal to the earth's, or, let us suppose, 200 radii, would be only $\frac{1}{8,000,000}$ of the earth's surface magnetic force in the corresponding position relatively to its magnetic axis. Considering, therefore (according to the principles explained by Mr. CHAMBERS), the sun as a magnet having its magnetic axis nearly perpendicular to the ecliptic, we see that, with an average intensity of magnetization equal to the earth's, the effect of reversing the sun's magnetization would be to introduce, in a direction perpendicular to the ecliptic, a disturbing force equal to about $\frac{1}{8,000,000}$ of the earth's average polar force, and would therefore be absolutely insensible to the most delicate terrestrial magnetic observation yet practised. A disturbing force of this amount, acting perpendicularly to the direction of the terrestrial magnetic force about the equator, would produce a disturbance in declination of only half a second; and the sun's magnetization would therefore need to be 120 times as intense as the earth's to produce a disturbance of $1'$ in declination even by a *complete reversal* in the most favourable circumstances. These estimates appear to me to give strong evidence in support of the conclusion at which Mr. CHAMBERS has arrived by a careful examination of the disturbances actually observed, that no effect of the sun's action as a magnet is sensible at the earth.

The same estimates are applicable to the moon, her apparent diameter being the same as the sun's. It is of course most probable that the moon is a magnet; but she must be a magnet thousands or millions of times more intense than the earth to produce any sensible effect of the character of any of the observed terrestrial magnetic disturbances.



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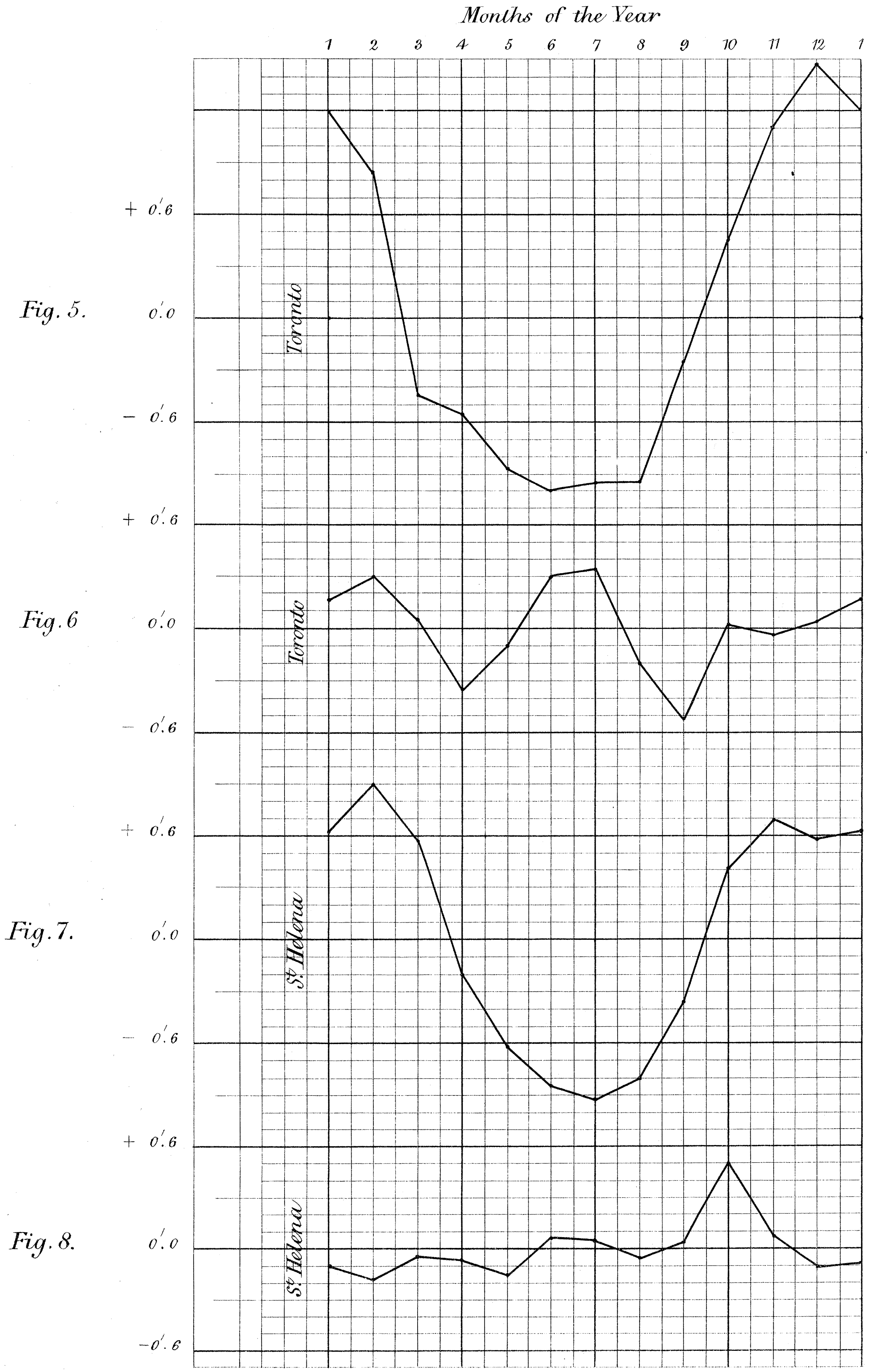

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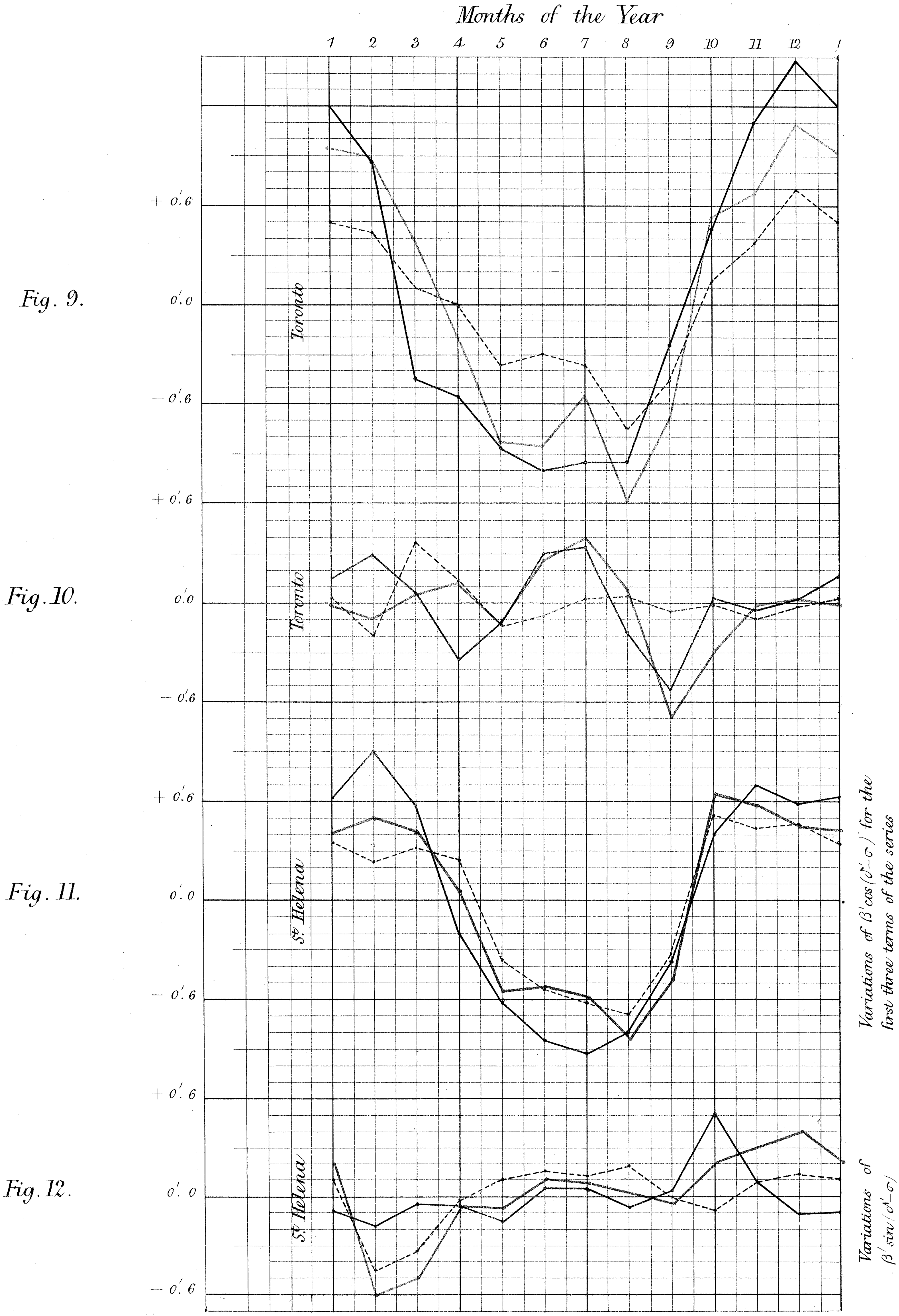

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Fig. 9.

Fig. 10.

Fig. 11.

Fig. 12.